Journal of Sound and Vibration (1997) **199**(4), 629–643

AN INVESTIGATION OF THE COUPLING LOSS FACTOR FOR A CYLINDER/PLATE STRUCTURE

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(*Received* 7 *July* 1995, *and in final form* 21 *May* 1996)

This paper presents a theoretical and experimental study of the transmission of vibration through a two element structure which consists of a cylindrical shell coupled to an end plate. The first part of this paper deals with the derivation of coupling loss factor (CLF) for the study of vibration transmission using Statistical Energy Analysis (SEA). The derivation is based on travelling wave analysis and the assumption of equipartition of energy amongst resonant modes. Numerical results for three examples of cylinder/plate structures are presented and compared with those of their equivalent plate/plate structures. The results show that the CLF values of the cylinder/plate structures asymptote to their equivalent plate/plate structures above the ring frequency of the cylinder. In the second part of this paper, an experimental program for measuring the CLF of an example cylinder/plate structure is described. Experimental results of the cylinder/plate structure show good agreement with theoretical predictions and confirm the validity of the present formulation of CLF.

1. INTRODUCTION

The analysis of noise transmission through complex systems often requires the prediction of structural vibration levels at high frequencies. Such predictions may be approached by using Statistical Energy Analysis (SEA) [1] which is a theoretical framework for analysing the average vibration levels of interconnecting subsystems based on energy flow relationships. The mean energy of the subsystems may be related to the input power by SEA parameters known as coupling loss factors (CLFs), internal loss factors and modal densities to form a set of linear, power balance equations. Solution of the power balance equations leads to the mean energy level of the individual subsystems.

Structures which consist of a number of coupled plate elements have been studied extensively using this approach [1–5] and good agreement between experimental results and SEA predictions reported. In contrast, the study of vibration transmission through coupled cylinder/plate structures has been less successful. Hwang and Pi [2] conducted an experimental investigation on a cylindrical shell welded onto a base plate and concluded that the SEA method was not capable of reaching any intelligent prediction of the coupling loss factor due to the strong interaction at the cylinder/plate interface. Blakemore *et al*. [6] studied a test structure which consisted of a number of flange-connected cylindrical

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shells and found considerable discrepancy between measurements and SEA predictions. They attributed the discrepancy to internal acoustic coupling, non-equipartition of energy between modes in a cylindrical shell element and low modal overlap. An experimental investigation of the CLFs of two cylinder/plate structures (one with a long thin cylinder and the other with a short squat cylinder) has been reported by Pollard [7] where he found conflicting results for the long cylinder although the short cylinder showed good agreement between the theoretical and experimental results. Recently, Schlesinger [8] presented a theoretical analysis of the transmission of vibration through a cylinder/plate coupled structure based on the distribution of wave energies in the radial, longitudinal and circumferential directions. The theory is supported by a limited amount of experimental data but further work is needed (both theoretical and experimental) to show that this method satisfies the reciprocity requirement of SEA.

The foregoing discussion indicates that a number of issues concerning the CLF of cylinder/plate structures have to be addressed. Reference [6] mentioned the use of wave transmission analysis to derive the CLF and presented theoretical predictions but no detailed derivation was given. Furthermore, reference [6] showed a considerable discrepancy between experimental results and theoretical predictions while results from references [2] and [7] were inconsistent with the principles of SEA. It should also be noted that the theoretical model presented in reference [8] requires further validation. Thus there is a need for a more detailed analysis, both theoretical and experimental, of the transmission of vibration through cylinder/plate coupled structures which are representative of many engineering structures.

This paper presents a theoretical analysis of the CLF for a thin cylindrical shell coupled to an end plate. The analysis is an extension of earlier work reported by the authors [9] which deals with the wave transmission properties of a number of cylinder/plate junctions. The formulation of CLF is based on the method of travelling wave analysis and the assumption of equipartition of energy amongst resonant modes. It should be noted that the concept of equipartition of modal energy is equivalent to a diffuse wave field for isotropic elements such as uniform plates [10] and a number of studies on coupled plate structures have been conducted to verify this assumption (see, for example, references [2], [3] and [5]). Thus the present study builds on the foundation of earlier work on coupled plate structures. Particular attention is paid to the reciprocity requirement in the derivation of CLF which is a fundamental principle of SEA. To verify the present formulation of CLF for a coupled cylinder/plate structure, an experimental program was conducted on a test structure to determine the distribution of vibrational energy due to a random input excitation. The power balance equations were then inverted to determine the CLF for comparison with theoretical predictions.

2. EVALUATION OF THE COUPLING LOSS FACTOR

2.1. COUPLING LOSS FACTOR

The CLF relates the amount of energy flow from one subsystem to another and may be derived from travelling wave analysis on the assumption that the wave field in each subsystem is diffuse [1]. The concept of a diffuse wave field poses no difficulty for isotropic elements like uniform flat plates but is less clear from an SEA point of view for non-isotropic elements like curved plates and cylinders. Langley [10] pointed out that the assumption of a diffuse wave field is equivalent to the equipartition of energy amongst the resonant modes for an isotropic element. He then derived the CLFs for structural junctions between curved plates based on the modal concept of equipartition of energy. In this section, the modal concept is extended to a cylindrical shell coupled to an end plate. The plate is assumed to have a hole cut out to accept the cylinder. This arrangement enables the results to be compared with those of an equivalent plate/plate structure.

Before proceeding to formulate the CLF, it is perhaps worthwhile to review briefly the wave propagation characteristics of a cylindrical shell. A cylindrical shell is subjected to three types of waves, often classified as Type I, II and III [11], and the behaviour of these waves depends strongly on the frequency of vibration. Above the ring frequency (the ring frequency is the frequency at which the wavelength of an extensional wave equals the mean circumference of the cylinder), the response of the cylinder is similar to that of a flat plate and the three types of waves in the cylinder (i.e., Type I, II and III) are therefore similar to the bending, shear and longitudinal waves respectively in a flat plate. However, the response of a cylinder below the ring frequency is strongly influenced by the effect of curvature which couples the cylinder displacements in the radial, circumferential and longitudinal directions. A measure of the degree of coupling of cylinder displacements is given by the amplitude ratios U/W and V/W, where U, V and W are the displacement amplitudes in the longitudinal, circumferential and radial directions. The amplitude ratios are functions of the frequency and the circumferential mode number n. A detailed discussion of the displacement characteristics of propagating waves in a cylindrical shell is given by Smith [11] where he shows that the Type I wave cuts on at a progressively higher frequency as the circumferential mode number is increased, together with lower amplitude ratios U/W and V/W. Thus the Type I wave has a higher radial component as the circumferential mode number is increased. Since the response of a cylinder may be considered as a superposition of each of the allowable circumferential modes at a particular frequency, it is argued that for a cylindrical shell having a response dominated by high order circumferential modes, the total response due to a Type I wave is dominated by the out-of-plane motion. The response of a thin cylindrical shell will be further discussed in section 3.2 where different methods for the measurement of loss factor are investigated.

When a cylindrical shell is coupled to an end plate, a Type I wave will generate bending and in-plane waves in the plate element. The significance of in-plane waves in the transmission of vibration has been investigated by Tratch [5] where he studied a number of coupled plate structures with different levels of complexity (from two to twelve coupled plates). He found that the in-plane waves act as ‘flanking paths’ for the bending motion and increase the energy transmission for complex structures which consist of more than two structural elements. However, for a simple structure with only two structural elements, as in the case for the present study, the in-plane waves generated in the elements have little effect on the flexural energy level.

It follows from the preceding discussion that the transmission of vibration through a cylinder/plate coupled structure is dominated by the out-of-plane motion and as a result, only such motion is considered in the present study. By modelling the cylindrical shell as a number of wave components representing each of the circumferential modes, the power loss by the cylinder due to coupling to the plate may be expressed as [12];

$$P_{cp} = (L_c/A_c) \sum_{n=0}^{N} E_{cn} c_{gcn} \tau_{cpn}, \qquad (1)$$

where L_c = coupling line length, A_c = surface area of cylinder, n = circumferential mode number, N = number of modes, E_{cn} = energy of the cylinder for the nth mode, c_{gcn} = group velocity of the cylinder for the nth mode and τ_{cpn} = transmission efficiency between the cylinder and plate for the nth mode when the cylinder is subjected to a Type I incident wave. Subscripts p and c refer to the plate and cylinder respectively. It should be noted that in-plane and out-of-plane waves with an angular dependency of the form cos $(n\theta)$ (θ is the polar co-ordinate of the plate) are generated in the plate due to an incident wave in the cylindrical shell.

The equipartition of subsystem energy amongst resonant modes implies that:

$$E_{cn} = E_c n(\omega)_{cn}/n(\omega)_c, \tag{2}$$

where E_c = total energy of the cylinder, $n(\omega)_c$ = modal density of the cylinder and $n(\omega)_{cn}$ = modal density of the cylinder for the nth circumferential mode.

Substituting equation (2) into (1) gives

$$P_{cp} = [L_c E_c/A_c n(\omega)_c] \sum_{n=0}^{N} c_{gcn} n(\omega)_{cn} \tau_{cpn}. \tag{3}$$

The transmitted power P_{cp} may be expressed in standard SEA form as:

$$P_{cp} = \omega \eta_{cp} E_c, \tag{4}$$

where η_{cp} is the CLF between the cylinder and plate. It follows from equations (3) and (4) that

$$\eta_{cp} = [L_c/\omega A_c n(\omega)_c] \sum_{n=0}^{N} c_{gcn} n(\omega)_{cn} \tau_{cpn}. \tag{5}$$

For a given circumferential mode, the number of resonance frequencies for a cylinder of length L is given by [13]:

$$N(\omega)_{cn} = L k_{cn}/\pi, \tag{6}$$

and hence the modal density

$$n(\omega)_{cn} = \partial N(\omega)_{cn}/\partial\omega = (L/\pi)\, \partial k_{cn}/\partial\omega = L/\pi c_{gcn}, \tag{7}$$

where k_{cn} is the axial wave number of the cylinder for the nth mode.

From equations (5) and (7), the CLF may now be expressed as:

$$\eta_{cp} = [1/\omega\pi n(\omega)_c] \sum_{n=0}^{N} \tau_{cpn}. \tag{8}$$

The transmission efficiency for a range of cylinder/plate coupled junctions τ_{cpn} has been evaluated by Tso and Hansen [9] and expressions for modal density of a cylindrical shell $n(\omega)_c$ may be obtained from the published literature (see, for example, reference [13]).

The concept of equipartition of energy amongst resonant modes in a subsystem presented in the preceding derivation of CLF may be shown to be equivalent to the assumption of a diffuse wave field in an isotropic system as follows. Consider two plates i and j coupled at right angles and subjected to a diffuse vibration field; the CLF of the structure is given by [12]:

$$\eta_{ij} = [c_{gi} L_c/\omega A_i 2\pi] \int_{-\pi/2}^{\pi/2} \tau_{ij} \cos\alpha \, \mathrm{d}\alpha = [c_{gi} L_c/\omega A_i \pi] \int_0^1 \tau_{ij} \, \mathrm{d}\,(\sin\alpha), \tag{9}$$

where α is the incident wave angle. If plate i is now simply supported along two parallel edges L_c apart (see Figure 1), the incident wave will consist of a component 'standing' in the y-direction (analogous to the circumferential mode) and a component 'propagating' in the x-direction (analogous to the axial mode of a cylinder). The incident wave angle may be expressed in terms of the wave number as

$$\sin\alpha = 2\pi n/L_c k_i \tag{10}$$

and equation (9) may now be written as:

$$\eta_{ij} = [c_{gi}L_c/\omega A_i \pi][2\pi/L_c k_i] \sum_{n=0}^{N} \tau_{ijn}. \tag{11}$$

By noting that the modal density for a two-dimensional system is given by:

$$n(\omega)_i = k_i A_i/2\pi c_{gi} \tag{12}$$

and substituting the modal density expression into equation (11) gives:

$$\eta_{ij} = [1/\omega\pi n(\omega)_i] \sum_{n=0}^{N} \tau_{ijn}, \tag{13}$$

which is identical to equation (8) for a cylinder/plate coupled structure derived under the assumption of equipartition of modal energy. A detailed discussion of this concept is presented in reference [1].

2.2. RECIPROCITY

One of the fundamental principles of SEA is that the CLFs must satisfy the requirement of reciprocity as stated below [1]:

$$n(\omega)_p \eta_{pc} = n(\omega)_c \eta_{cp}. \tag{14}$$

Consider a cylindrical shell coupled to an annular plate with the latter subjected to an out-of-plane incident wave with a circumferential dependency of $\cos(n\theta)$ propagating radially towards the cylinder/plate interface. It can be shown from the asymptotic expansion of the Bessel functions (see, for example, reference [14]) that the wave amplitude is inversely proportional to the square root of the radius. Since the energy is proportional

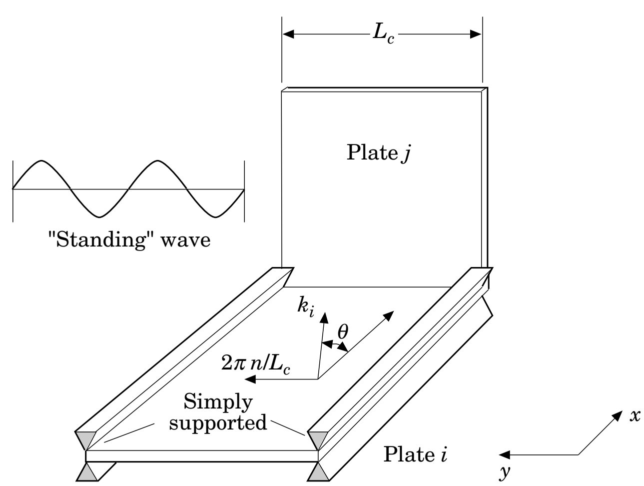

Figure 1. Plate/plate junction.

to the square of the wave amplitude, the energy density of the plate for a given circumferential mode number n may be expressed as:

$$e_n \propto 1/r, \qquad \text{or} \qquad e_n = K/r, \tag{15}$$

where K is a constant and r is the radius. The energy of the plate can be obtained by integrating the energy density over the entire plate area:

$$E_{pn} = \int_a^b (K/r) 2\pi r \, \mathrm{d}r = 2\pi K(b - a), \tag{16}$$

where b and a are the outer and inner radii of the plate respectively. From equations (15) and (16), the energy density of the plate at the plate/cylinder interface is given by:

$$e_n = E_{pn}/[2\pi a(b - a)]. \tag{17}$$

The wave number of the incident wave propagating in the radial direction at a circumferential mode number n may be obtained from the asymptotic expansion of the Bessel functions [14]:

$$k_{pn} = k_p - n\pi/2r - \pi/4r, \tag{18}$$

for $k_p r \gg 1$ and $k_p r \gg n^2$; where k_p is the wave number of the plate. It follows from equation (18) that the group velocity of this wave may be expressed as:

$$c_{gpn} = 1/[\partial k_{pn}/\partial \omega]. \tag{19}$$

The power transmitted from the plate to the cylinder may now be expressed in terms of the energy density of the plate as:

$$P_{pc} = L_c \sum_{n=0}^{N} e_n \tau_{pcn} c_{gpn}, \tag{20}$$

From equations (17) and (20)

$$P_{pc} = [1/(b - a)] \sum_{n=0}^{N} E_{pn} \tau_{pcn} c_{gpn}. \tag{21}$$

By using the assumption of equipartition of energy and the standard SEA expression for CLF (see equations (2) and (4)), the CLF between the plate and cylinder is given by:

$$\eta_{pc} = [1/\omega n(\omega)_p (b - a)] \sum_{n=0}^{N} c_{gpn} n(\omega)_{pn} \tau_{pcn}. \tag{22}$$

For a given order of nodal diameter n, the modal density of the plate may be written as [13]:

$$n(\omega)_{pn} = \partial N(\omega)_{pn}/\partial \omega = [(b - a)/\pi] \, \partial k_{pn}/\partial \omega = (b - a)/\pi c_{gpn}. \tag{23}$$

Substituting equation (23) into (22) leads to the following expression for CLF:

$$\eta_{pc} = [1/\omega \pi n(\omega)_p] \sum_{n=0}^{N} \tau_{pcn}. \tag{24}$$

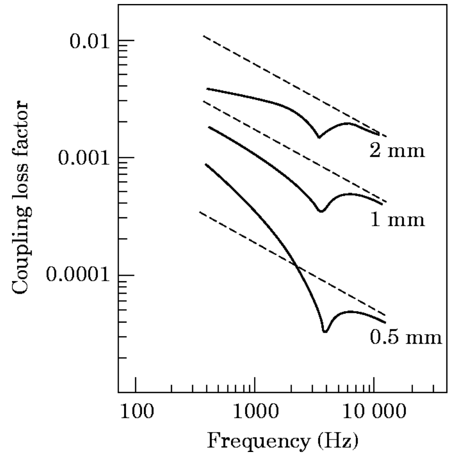

Figure 2. Coupling loss factor for different cylinder/source plate thicknesses. ——, cylinder/plate; – – –, plate/plate.

By comparing equation (8) with equation (24) and noting that the transmission efficiency is symmetric ($\tau_{cpn} = \tau_{pcn}$), it can be seen that the reciprocity requirement is satisfied.

2.3. NUMERICAL EXAMPLES

Calculations were performed on the CLFs of three steel cylinders each coupled to a 2 mm thick steel end plate. The shell thicknesses of the three cylinders were 0·5, 1·0 and 2·0 mm respectively. The length and mean diameter of all cylinders were chosen to be 0·8 m and 0·45 m respectively to coincide with an example structure for an experimental investigation described in section 3. Figure 2 shows the CLFs of the three cylinder/plate structures. The CLFs of their corresponding equivalent plate/plate structure based on a diffuse bending wave field are also plotted in Figure 2 for comparison. The source plates are assumed to have the same surface area and thickness as those of their corresponding cylindrical shells.

It can be seen from Figure 2 that all of the cylinder/plate structures show a dip in the CLF at around the ring frequency of 3730 Hz, presumably caused by the increase in modal density of the cylinder around the ring frequency region. Thereafter the CLFs asymptote to the values of their equivalent plate/plate structures as the frequency increases. This finding is consistent with the well established fact that the response of a cylinder approaches that of a flat plate at high frequencies (above the ring frequency). Below the ring frequency, the response of a cylinder is dominated by the membrane effects and as a result, the CLFs of the cylinder/plate structures differ considerably from their equivalent plate/plate structures.

3. EXPERIMENTAL INVESTIGATION

3.1. EXPERIMENTAL ARRANGEMENT

An experimental program has been designed to verify the CLF for a cylinder/plate coupled structure developed in section 2.1. The test structure consists of a thin cylinder and an end plate as shown in Figure 3. Steady state power balance measurements [3] and reverberation time measurements were conducted on the individual cylinder and plate elements to determine their internal loss factors. The elements were then welded together to form a rigid connection and further tests conducted to determine the distribution of vibrational energy due to a random input excitation. The application of welding to form a rigid connection between the structural elements is consistent with previous experimental studies on the transmission of vibration in coupled structures (see, for example, references [5] and [7]). To control the dissipation of heat during the welding process, it was decided to run a small section of weld at a time and alternate the process at different parts of the joint in order to maintain an even distribution of heat across the plate and shell elements. These measures minimise the distortion of the elements and the effect of heat on the damping material. The effect of welding on the response and damping of the structural elements is investigated in section 3.2, where the internal loss factors of the individual elements are compared with *in-situ* measurements.

The frequency range of the experiment was selected to be 500–8000 Hz, corresponding to a ring frequency ratio Ω of 0·11–2·14. This enabled the effects of cylinder curvature on vibratory power transmission to be investigated at low frequencies ($\Omega \ll 1$). Also, at the higher end of the frequency spectrum ($\Omega > 2$), the cylinder is expected to behave approximately as a flat plate and well established results on plate/plate structures may be used to check against the present theory. In line with previous research work on the transmission of vibration through coupled cylindrical structures [2, 6, 7] and the discussion presented in section 2.1, the present experimental investigation is limited to the out-of-plane motion of the cylinder and plate. To check that the in-plane motion has no significant effect on the flexural energy level, the modal density of the plate element subjected to an in-plane motion was calculated and compared with the out-of-plane modal density. The in-plane modal density was found to be $4{\cdot}02 \times 10^{-5}$ and $6{\cdot}43 \times 10^{-4}$ s/rad at a frequency of 500 and 8000 Hz respectively, compared with an out-of-plane modal density of $1{\cdot}52 \times 10^{-2}$ s/rad (independent of frequency). Thus the plate energy is dominated by the out-of-plane motion. Both the cylinder and plate have more than 10 resonant modes (out-of-plane) in the lowest third octave band. Calculations performed on the dispersion equation of the cylindrical shell show that 10 circumferential modes exist at a frequency of 500 Hz.

Consideration was given to the damping requirement for the cylinder and plate. If the cylinder/plate structure has a CLF very much greater than the internal loss factor of the individual structural elements, then the modal energy of the elements would be approximately equal and insensitive to variations in the CLF. To determine the CLF from energy and power measurements, it is therefore desirable to have the internal loss factors

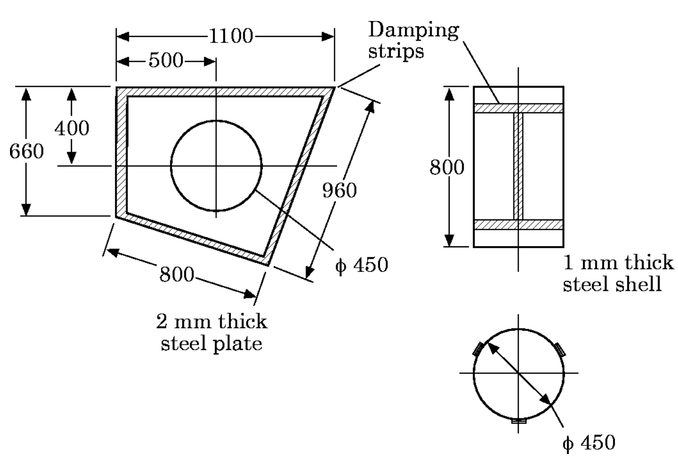

Figure 3. Cylinder and plate elements. All dimensions in mm.

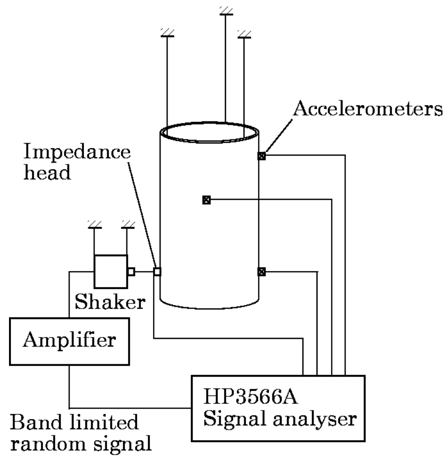

Figure 4. Experimental setup for steady state power balance measurement.

at least the same order of magnitude as that of the CLF. For the present study, this was achieved by adding self-adhesive damping strips to the cylinder and plate (see Figure 3).

Figure 4 shows the set-up of the experiment for steady state power balance measurements. The structure was suspended by strings and driven by an electromagnetic shaker through an impedance head. Care was taken to align the shaker axis normal to the test structure and a thin stinger was used to connect the shaker to the impedance head to minimise the input of bending moment and in-plane force. Band limited random signals were used as the excitation source.

Another point of consideration for the present experimental study was the excitation source. Fahy [15] has suggested that the injection of power to a structure with point excitation will result in modes which are not statistically independent (i.e., coherent modes) and violate a basic assumption used in SEA modelling. Bies and Hamid [3] have studied this problem and showed that modal incoherence can be achieved by averaging the results over three randomly chosen excitation points. The present study followed this approach in the measurement of internal loss factor using the steady state power balance method and the distribution of vibrational energy for the coupled structure. In the latter measurement, the cylinder was first excited and measurements taken to determine the input power and vibrational energy of the cylinder and plate. The experiment was then repeated by injecting power into the plate element to check for reciprocity. To determine the power injected into the structural element, the force and acceleration signals from the impedance head were processed using the following expression:

$$P = 1/2 \text{ Re } \{F \times (A/j\omega)^*\}, \tag{25}$$

where F and A are the complex amplitudes of the force and acceleration respectively, and * denotes the complex conjugate. The vibrational energy was determined from the spatial average of the acceleration signals from a number of randomly chosen points on each element. To determine the number of measurement points necessary for an accurate estimation of the spatially averaged response, preliminary tests were conducted on each element by using six and eight accelerometers in turn for spatial averaging. It was found that both test configurations resulted in approximately the same value of spatially averaged acceleration and henceforth six accelerometers were used to evaluate the vibrational energy of each of the elements.

Before calculating the vibrational energy, the acceleration signals were corrected to allow for the effect of mass loading using the following expression [16]:

$$|A_0/A_m|^2 = 1 + |\omega M_a/Z|^2, \tag{26}$$

where A_0 = complex amplitude of the acceleration of the unloaded structure, A_m = complex amplitude of the acceleration measured by the accelerometer, M_a = accelerometer mass and Z = impedance of the test structure.

The vibrational energy is then given by:

$$E = 1/2M\overline{|A_0/j\omega|^2}, \tag{27}$$

where M is the mass of the structural element and $^-$ denotes the spatial average.

3.2. RESULTS

The internal loss factor of a structural element is related to the input power P_i and vibrational energy E_i through the expression

$$\eta_i = P_i/E_i\omega. \tag{28}$$

Alternatively, the internal loss factor may be determined from the reverberation time method using the expression:

$$\eta_i = 2{\cdot}2/f\,T_{60}, \tag{29}$$

where f is the frequency and T_{60} the reverberation time.

Experiments were carried out on the cylinder and plate using both methods of measurement. In the steady state power balance method as described in section 3.1 above, the results were averaged over three randomly chosen excitation points. A hammer impact was used as the excitation for the reverberation time method. The time history of the acceleration signal was recorded after passing through a third octave filter and subsequently processed to obtain a complex signal with the real and imaginary parts given by the measured acceleration and its Hilbert Transform [17] respectively. The magnitude of this complex signal was then calculated and plotted on a logarithmic scale for the estimation of reverberation time. Five averages of the time history were taken for each measurement point and the internal loss factor was averaged over three randomly chosen measurement points. A typical envelope of the acceleration time history is shown in Figure 5.

Figures 6 and 7 show the internal loss factor of the cylinder and plate respectively. It can be seen that the results given by these two methods are in reasonable agreement. However, due to the coupling of in-plane and out-of-plane motions in the cylinder, the results for the cylinder internal loss factor have to be interpreted carefully. For the reverberation time method, the internal loss factor is related to the decay of the out-of-plane motion. This is not the case for the steady state power balance method since the energy dissipated includes both in-plane and out-of-plane motions. Coupling these two types of motion will result in in-plane motion which is not measured by the accelerometer at the power injection point. However, there is no in-plane external power input into the cylinder provided that the external excitation transmits no in-plane force or moment into the cylinder. This condition was met in the present experimental study by mounting the axis of the shaker normal to the cylinder and using a thin stinger (1 mm in diameter by 40 mm in length) to attach the shaker to the impedance head which in turn was bonded directly to the cylinder. On the other hand, the energy in the cylinder is distributed into components associated with both in-plane and out-of-plane motions. The latter motion was damped in the present experiment by self-adhesive damping strips which were arranged to give an effective damping for both the circumferential and axial modes.

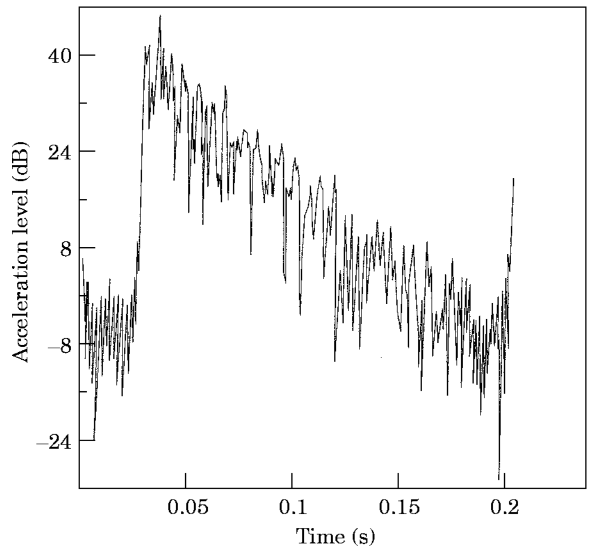

Figure 5. Typical decay record for cylindrical shell. Third octave band at a band centre frequency of 1·25 kHz.

Referring back to Figure 6 which shows the internal loss factor of the cylinder using the steady state power balance method and the reverberation time method, an agreement between these two methods of measurement suggests that the input power to the cylinder is predominantly dissipated by the out-of-plane motion and justifies the present experimental investigation where only such motion is considered. This is expected from the theoretical consideration in section 2.1 since the cylindrical shell has a high order of circumferential modes.

Subsequent to the joining of the cylinder to the end plate, measurements were carried out to determine the input power and the vibrational energy distribution by first exciting the cylinder and then repeating the experiment by exciting the plate. Before determining the CLF from the power and energy measurements of the coupled structure, the results were checked for reciprocity. Clarkson and Ranky [4] have shown that the reciprocity requirement is satisfied when

$$[E_{ij}/P_j n(\omega)_i]/[E_{ji}/P_i n(\omega)_j] = 1, \qquad (30)$$

where E_{ij} is the energy of element i due to an input excitation at element j.

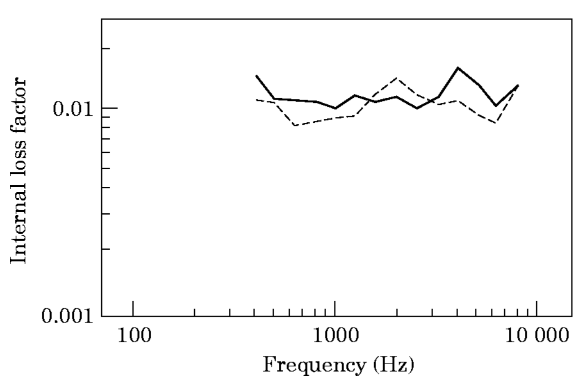

Figure 6. Internal loss factor of cylinder. ——, steady state power balance method; – – –, reverberation decay technique.

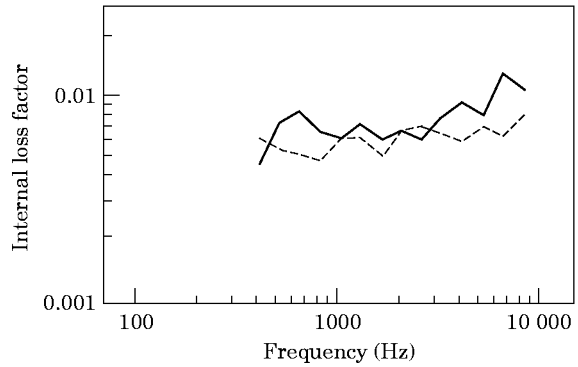

Figure 7. Internal loss factor of plate. Key as Figure 6.

The left hand side of equation (30) representing the energy ratio is plotted in Figure 8. It can be seen that the energy ratio is reasonably close to unity, and thus the results satisfy the requirement for reciprocity. A matrix inversion routine based on the minimisation of the sum of the squared errors [3] was used to determine the internal loss factors of the cylinder and plate, as well as the CLF of the structure. This method involves a re-arrangement of the energy balance equations in the following form:

$$\Delta_1 = \eta_1 E_{11} + \eta_{12} E_{11} - \eta_{21} E_{12} - P_1/\omega, \tag{31}$$

where Δ_1 denotes the experimental errors in determining the power and vibrational energy. Similar equations may be formulated for other subsystems and for the present experimental set up, a total of four equations were formulated (two subsystems for each configuration of input excitation). The sum of the squared errors may then be expressed as:

$$S = \sum_{i=1}^{4} \Delta_i^2. \tag{32}$$

Following the least square procedure, the sum of the squared errors may be minimised with respect to the internal loss factors and CLFs:

$$\partial S/\partial \eta = 0. \tag{33}$$

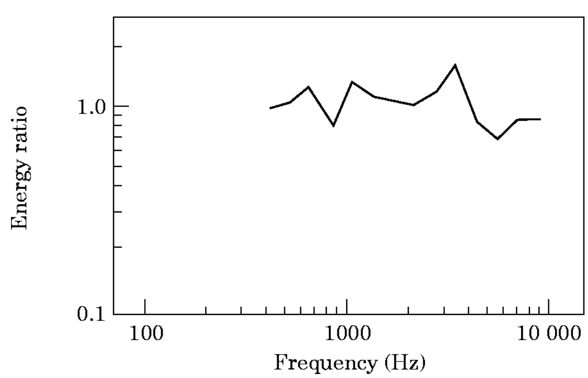

Figure 8. Energy ratio of the coupled cylinder/plate structure.

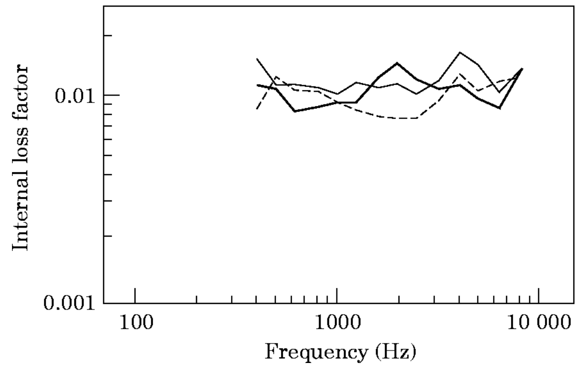

Figure 9. Internal loss factor of cylinder. ——, steady state power balance method; – – –, *in-situ* method; ═══, reverberation decay technique.

By using the reciprocity condition (i.e., $n(\omega)_p \eta_{pc} = n(\omega)_c \eta_{cp}$), equation (33) constituted three linear algebraic equations and the internal loss factors and CLF were determined by standard matrix inversion of these equations.

Figures 9 and 10 show the internal loss factor of the cylinder and plate respectively. The results given by the *in situ* method (inversion of matrix) are in close agreement with the steady state power balance method. This finding is consistent with earlier work reported by Bies and Hamid [3] for the case of flat plates and supports the present experimental approach in determining the CLF. It also suggests that the coupled modal energies of the elements are approximately equal to the uncoupled modal energies. Furthermore, the welding process has little effect on the damping of the structural elements. Figure 11 shows the CLFs of the cylinder/plate structure obtained by equation (8) and measurement. The experimental results are fairly well predicted by the theory presented in section 2.1 although the theoretical values are slightly higher than the measured values in the frequency range of 800–2500 Hz. The dip in CLF which is predicted in the theoretical analysis can be observed in the experimental data. It occurs at a frequency of around 2500 Hz compared with a predicted value of 3730 Hz which corresponds to the ring frequency of the cylinder. The experimental data also shows some discrepancy with the predicted CLF above a frequency of 6300 Hz. An attempt to conduct further tests (above 8000 Hz) to confirm the convergence of the experimental results to the theoretical CLF of a plate/plate structure was hampered by the limitation in sampling rate of the data acquisition system. However, further examination of the results reveals that the

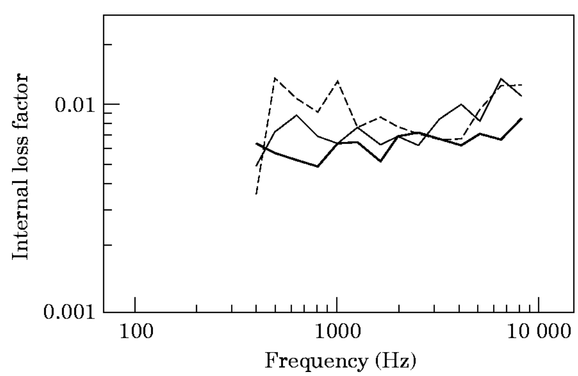

Figure 10. Internal loss factor of plate. Key as Figure 9.

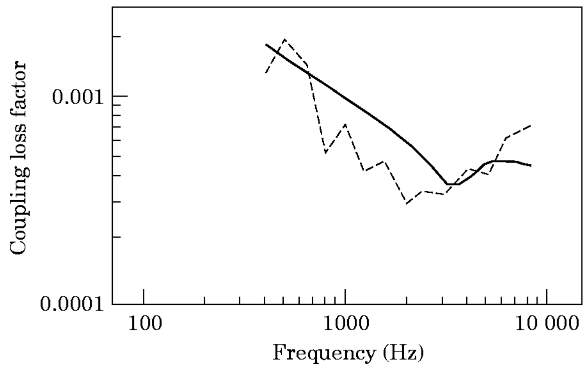

Figure 11. Coupling loss factor of cylinder/plate structure. ——, theory; – – –, experiment.

discrepancy is consistent with previous work on the experimental investigation of CLF (see, for example, references [3] and [4]) and may be partially attributed to the random nature of the experiment as well as the assumptions involved in the derivation of CLF (for example, the equipartition of energy amongst circumferential modes). Overall, the calculated CLF is considered to be satisfactory as an SEA parameter for the estimation of response levels in a cylinder/plate structure.

4. CONCLUSIONS

A theoretical model for evaluating the CLF of a cylindrical shell coupled to an end plate has been presented. It has been demonstrated that the condition of reciprocity is satisfied by using a cylinder/annular plate coupled structure. From calculations of the CLFs of three cylinder/plate structures, it was found that the CLF values asymptote to those of their equivalent plate/plate structures above the ring frequency. The calculated CLF has been verified by experiment and considered to be satisfactory as an SEA parameter.

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